

# Effect of Coulomb scattering on graphene conductivity

V. Vyurkov<sup>1\*</sup> and V. Ryzhii<sup>2†</sup>

<sup>1</sup>*Institute of Physics and Technology RAS, Moscow 117218, Russia*

<sup>2</sup>*University of Aizu, Aizu-Wakamatsu 965-8580, Japan,  
and Japan Science and Technology Agency, CREST, Tokyo 107-0075, Japan*

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The effect of Coulomb scattering on graphene conductivity in field effect transistor structures is discussed. Inter-particle scattering (electron-electron, hole-hole, and electron-hole) and scattering on charged defects are taken into account in a wide range of gate voltages. It is shown that an intrinsic conductivity of graphene (purely ambipolar system where both electron and hole densities exactly coincide) is defined by strong electron-hole scattering. It has a universal value independent of temperature. We give an explicit derivation based on scaling theory. When there is even a small discrepancy in electron and hole densities caused by applied gate voltage the conductivity is determined by both strong electron-hole scattering and weak external scattering: on defects or phonons. We suggest that a density of charged defects (occupancy of defects) depends on Fermi energy to explain a sub-linear dependence of conductivity on a fairly high gate voltage observed in experiments. We also eliminate contradictions between experimental data obtained in deposited and suspended graphene structures regarding graphene conductivity.

Different heterostructures on the basis of *Graphene*, i.e., a monolayer of carbon atoms forming a dense honeycomb two-dimensional crystal structure are considered as promising candidates for future micro- and nanoelectronics. (see [1, 2, 3, 4, 5] and references therein). The features of the electron and hole energy spectra in graphene provide the exceptional properties of graphene-based heterostructures and devices, in particular, field-effect transistors [4, 6, 7, 8]. Graphene band structure peculiarities have a substantial impact on Coulomb scattering: inter-particle scattering (electron-electron, hole-hole, and electron-hole) and scattering on charged defects. Here we intend to elucidate the effect of that kind of scattering on graphene conductivity taking into account either deposited and suspended graphene structures.

Experimental studies of graphene have revealed a quite amazing feature of its conductivity. There is almost no dependence of the minimum conductivity of graphene  $\sigma_{min}$  on temperature (between 0.3K and 300K) in deposited graphene structures [3] (Fig.1, upper panel). This is really intriguing because at the same time the carrier density varies in this temperature interval by six orders of magnitude. The minimum conductivity (or intrinsic conductivity) arises at zero gate voltage in gated graphene structures. Hereafter, as usual, we reference the zero gate voltage ( $V_G = 0$ ) just to this point. The conductivity is approximately equal to  $\sigma_{min} \simeq (6 k\Omega)^{-1}$ . There appears a natural temptation to bind this value to two conductance quanta [3, 9]. The conductance quantum for spin-unpolarized electrons equals  $G_0 = 2e^2/h = (12k\Omega)^{-1}$ , where  $e$  is the electron elementary charge and  $h$  is the Planck constant.

The minimum conductivity of graphene corresponds

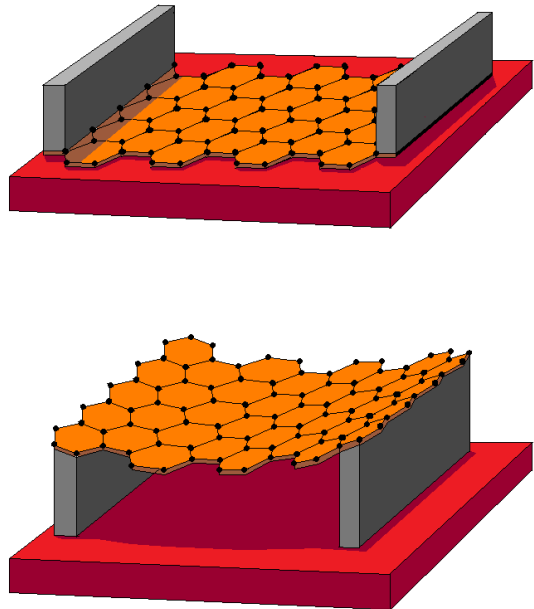


FIG. 1: Schematic view of graphene structures with two side contacts and a wafer as a back gate: deposited graphene (upper panel) and suspended graphene (lower panel)

to an utterly ambipolar system when electron density  $\Sigma_e$  and hole density  $\Sigma_h$  coincide:  $\Sigma_e = \Sigma_h$ . In the situation, the electron-hole scattering is the strongest scattering mechanism. We propose below a derivation based on a scaling theory approach (see also [10]).

The conductivity,  $\sigma_0$ , of an intrinsic graphene layer determined by the Coulomb scattering of electrons and holes can be expressed as follows:

\*Electronic address: vyurkov@ftian.ru

†Electronic address: v-ryzhii@u-aizu.ac.jp

$$\sigma_0 = \frac{k_{eff}^2}{e^2} f_0, \quad (1)$$

where  $k_{eff}$  is the effective permittivity and  $f_0$  is a so far unknown function of the parameters of the graphene electron-hole system and, generally speaking, of the temperature. The proper dependence (1) on the elementary charge  $e$  and the effective permittivity  $k_{eff}$  for the Coulomb interaction in the Born approximation is factorized. As it was recently theoretically demonstrated [11, 12, 13, 14], the screening in a graphene layer, turns out to be determined by a weak function of the dimensionless parameter

$$\alpha = \frac{e^2}{k_{eff} \hbar v_F}, \quad (2)$$

where  $\hbar$  is the reduced Planck constant. In reality, the screening by highly conducting gates might be substantial were they placed fairly close to a graphene sheet.

The factor  $f_0$  in (1) can solely depend on intrinsic graphene parameters and the Plank constant. In the case when electron and hole densities coincide  $\Sigma_e = \Sigma_h = \Sigma_T$ , the graphene electron-hole system is fully characterized by the only parameter besides Fermi velocity  $v_F$ . It could be the mean momentum  $p_T$ , mean wave vector  $k_T = p_T/\hbar$ , temperature  $T \simeq \hbar v_F k_T/k$  (where  $k$  is the Boltzmann constant), or density  $\Sigma_T \simeq k_T^2$ . All those parameters are directly bound to each other. Therefore, the only possible combination looks like

$$\sigma_0 = \frac{\kappa_{eff}^2 \hbar v_F^2}{e^2} = \frac{2\kappa_{eff}^2 v_F^2}{G_0}. \quad (3)$$

Here we have neglected a numerical factor of the order of unity, which could be derived via thorough consideration of electron-hole scattering [15] or determined from experimental data (see, for instance, [3]).

The only temperature-dependent parameter  $k_T$  describing the graphene electron-hole system can not be included. Therefore, the graphene conductance appears to be independent of the temperature. Moreover, it is readily seen that the graphene conductivity turned out to be inversely proportional to the conductance quantum  $G_0 = 2e^2/h = (13k\Omega)^{-1}$ , in spite of the previous expectations [3].

It is expedient to estimate the value of  $\sigma_0$  for the typical graphene structures. When a graphene sheet is sandwiched between two dielectrics with permittivity  $k_1$  and  $k_2$ , the effective permittivity is

$$\kappa_{eff} = \frac{\kappa_1 + \kappa_2}{2}. \quad (4)$$

Therefore, in our opinion, the crucial experiment to verify the above speculations lies in changing the permittivity of the adjacent dielectrics. In most of so far made

experiments a graphene layer was deposited on a SiO<sub>2</sub> surface. The effective permittivity in this case is equal to  $k_{eff} \simeq 2.5$ . The velocity  $v_F = 10^8$  cm/s corresponds to the conductance equal to  $(3k\Omega)^{-1}$ . Substituting all that into (3), one obtains  $\sigma_0 \simeq (4k\Omega)$ . This is a very good agreement with the experimental value of  $(6k\Omega)^{-1}$  [3] provided a numerical coefficient in (3) was ignored.

All said above is related to an intrinsic graphene conductivity when both the electron and hole densities are exactly equal to each other, i.e.,  $\Sigma_e = \Sigma_h = \Sigma_T$ . This can also occur in graphene-based structures with a highly conducting gate similar to field-effect transistors. In such a case, the graphene conductivity is determined, in fact, by the friction between the electron and hole subsystems. It appears to be interesting to apprehend what occurs beyond this equality which can be broken by an applied gate voltage. At a moderate gate voltage there is a rather small difference in the electron and hole densities:  $|\Sigma_e - \Sigma_h| \ll \Sigma_e, \Sigma_h$ , so that the electron-hole scattering mechanism is still the strongest one. However, we argue that in this case, the behavior of the conductivity is governed by both the electron-hole as well as other scattering processes, for instance, those on phonons and charged defects. In contrast to an intrinsic graphene, the influence of the electron-hole scattering on the conductivity becomes intermediate. Due to this scattering processes, the majority carriers drag the minority ones. In this case the electron-hole system could be regarded as a unified fluid with a total density equal to  $\Sigma = \Sigma_e + \Sigma_h$  and a renormalized elementary charge of composing particles given by the following equation:

$$\tilde{e} = e \frac{\Sigma_h - \Sigma_e}{\Sigma_h + \Sigma_e}. \quad (5)$$

Taking into account (5), the conventional formula for the conductivity of a two-dimensional electron gas can be adopted to the conductivity of graphene:

$$\sigma_1 = \frac{\tilde{e}^2 \Sigma}{\nu_{ex} M} = \frac{e^2}{\nu_{ex} M} \frac{(\Sigma_e - \Sigma_h)^2}{(\Sigma_e + \Sigma_h)}, \quad (6)$$

where  $\nu_{ex}$  is the rate of scattering on phonons and defects,  $M$  is the hydrodynamic mass of particles arising in the hydrodynamic equations for graphene electron-hole system. The latter can be roughly estimated as

$$M \simeq \frac{\hbar k_T}{v_F} \simeq \frac{\hbar \sqrt{\Sigma_T}}{v_F}. \quad (7)$$

The conductivity (6) implies a quadratic dependence on gate voltage  $\sigma_1 \propto V_G^2$  (Fig. 2, low voltage) as a sheet charge  $\Sigma_e - \Sigma_h \propto V_G$ . A crossover from intrinsic graphene conductivity  $\sigma_0$  given by formula (3) to that given by formula (5) can be very fast because external scattering  $\nu_{ex}$  is much weaker compared to electron-hole scattering  $\nu_{eh}$  which could be easily extracted from the expression (3). If among external scattering mechanisms phonon

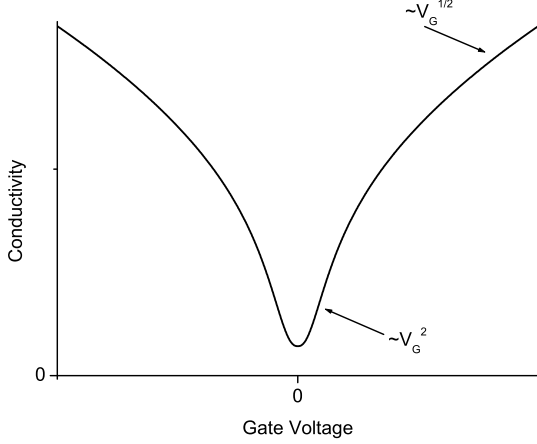


FIG. 2: Graphene conductivity vs. gate voltage.

scattering dominates there appears a definite dependence of conductivity on temperature. Obviously, the phonon scattering can predominate and determine conductivity of perfect graphene structures [16]. Then, according to estimations, the electron and hole mobility can achieve values above 200 000 cm<sup>2</sup>/V s instead of nowadays 5 000 - 15 000 cm<sup>2</sup>/V s at room temperature for deposited graphene structures (Fig. 1, upper panel) [17].

Recently a conductivity of suspended graphene (Fig. 1, lower panel) was experimentally investigated [18]. This is just a structure where phone scattering can prevail over defect scattering. For the first time the authors observed temperature dependent minimum graphene conductivity. However, we attribute this unusual behavior to the fact that suspended graphene was bent and, therefore, inhomogeneous with respect to charge distribution. The authors have admitted this possibility. In other words, there were regions obeying the relation (3) and that obeying the relation (5) simultaneously. Very high mobility (100 cm<sup>2</sup>/V s) observed close to the point of intrinsic graphene when electron and hole densities almost coincide, really, confirms that phonon scattering is dominant with respect to that on defects in the case. Nevertheless, below we demonstrate that at high voltage a proportion of phone scattering rate to defect scattering rate can become reciprocal.

At fairly high gate voltages when  $\Sigma_e \gg \Sigma_h$  or vice versa, the graphene electron-hole system is virtually unipolar and the electron-hole scattering processes are not essential. In that case, the conductivity of the graphene structures fabricated so far is likely governed by the scattering on charged defects [3, 14, 16], as there is no temperature dependence of mobility evidenced by experiments. Nevertheless, theoretical attempts to investigate an interplay of phonon scattering and Coulomb scattering among particles are made [19].

The conductivity  $\sigma_2$  limited by the scattering on

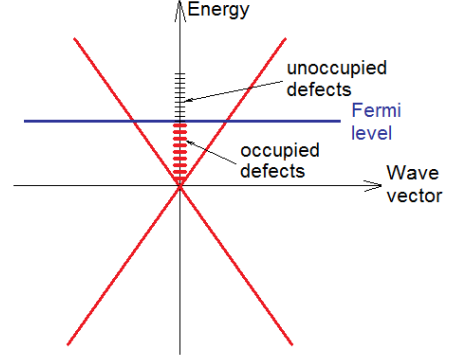


FIG. 3: Charged defect states under the Fermi level.

charged defects can be obtained almost similarly as that for the case of the dominant electron-hole scattering. Following (1), the conductivity in this situation can be presented as

$$\sigma_2 = \frac{k_{eff}^2}{e^2 \Sigma_i} f_2, \quad (8)$$

where  $\Sigma_i$  is the density of charged defects. If the thermal energy  $kT$  is much smaller than the Fermi energy  $\varepsilon_F$ , the factor  $f_2$  in (8) can depend only on the majority carrier density  $\Sigma \simeq \Sigma_e$  (or  $\Sigma \simeq \Sigma_h$ ), the Fermi velocity  $v_F$ , and the Plank constant  $h$ , so that

$$\sigma_2 = \frac{k_{eff}^2 h v_F^2 \Sigma}{e^2 \Sigma_i}. \quad (9)$$

For an invariable density of defects, (9) results in a linear dependence of the conductivity on the gate voltage. On the contrary, it seems plausible to suggest that the charged defect concentration can depend on position of the Fermi level in graphene, i.e., on the Fermi energy  $\varepsilon_F$  (Fig.3).

For instance, such defects may arise at the graphene-SiO<sub>2</sub> interface. If the number of defects per unit energy and area is constant and equals  $n_i$ , their sheet density is

$$\Sigma_i = n_i \varepsilon_F \simeq n_i v_F \hbar \sqrt{\Sigma}. \quad (10)$$

After substituting (10) into (9), one can arrive at a square-root dependence of the conductivity on the gate voltage  $\sigma_2 \propto \sqrt{V_G}$  (Fig.2, high voltage). The experimental data show that the observed voltage dependences are somewhat in between the linear and square-root dependences [17]. In the case of an intrinsic graphene, the Fermi energy in (10) should be replaced by the thermal energy  $kT$ . Since according to (10), the density of charged defects can be smaller than that of carriers, the electron-hole scattering mechanism can really dominate even at low temperatures.

In conclusion, we have asserted that an intrinsic graphene conductivity (when both electron and hole densities exactly equal to each other) is defined by electron-hole scattering. It has a universal value independent of temperature. We have given an explicit derivation based on scaling theory. This value could be manipulated by varying permittivity of surrounding dielectrics. When there is even a small deviation in electron and hole densities caused by applied gate voltage the situation becomes utterly different. In the case the conductivity is deter-

mined by both strong electron-hole scattering and weak external scattering: on defects or phonons. If phonon scattering prevails the conductivity depends on temperature. We also suggest that density of charged defects (occupancy of defects) depends on Fermi energy to explain a sub-linear dependence of conductivity on gate voltage observed in experiments.

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